EDEXCEL CORE2 JUN 2009

1)
$$\int_1^4 (2x + 3\sqrt{x})\,dx$$
$$= \int_1^4 (2x + 3x^{\frac{1}{2}})\,dx$$
$$= \left[\frac{2x^2}{2} + \frac{3x^{3/2}}{3/2}\right]_1^4$$
$$= \left[x^2 + 2x^{3/2}\right]_1^4$$
$$= (4^2 + 2\times 4^{3/2}) - (1^2 + 2\times 1^{3/2})$$
$$= (16 + 2\times 8) - (1 + 2\times 1)$$
$$= 32 - 3$$
$$= 29$$

2) a)
$$(2 + kx)^7$$
$$= 2^7 + \binom{7}{1} 2^6 (kx) + \binom{7}{2} 2^5 (kx)^2 + +$$
$$= 128 + 448kx + 672k^2x^2 + +$$

b) Given that $672k^2 = 6(448k)$
$$672k^2 = 2688k$$
$$\Rightarrow k = \frac{2688}{672}$$
$$\Rightarrow k = 4$$

3) $f(x) = (3x-2)(x-k) - 8$

a)
$$f(k) = (3k-2)(k-k) - 8$$
$$= 0 - 8$$
$$= -8$$

3b) By remainder theorem the remainder when $f(x)$ is divided by $(x-2)$ is $f(2)$

$\therefore f(2) = 4$

$$f(2) = (3(2)-2)(2-k) - 8 = 4$$
$$4(2-k) - 8 = 4$$
$$8 - 4k - 8 = 4$$
$$-4k = 4$$
$$k = \frac{4}{-4}$$
$$k = -1$$

$$f(x) = (3x-2)(x+1) - 8$$
$$= 3x^2 - 2x + 3x - 2 - 8$$
$$= 3x^2 + x - 10$$
$$= (3x-5)(x+2)$$

$$f(x) = (3x-5)(x+2)$$

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4a)

x	$\sqrt{2^x+1}$
0	1.414
0.5	1.554
1	1.732
1.5	1.957
2	2.236
2.5	2.580
3	3

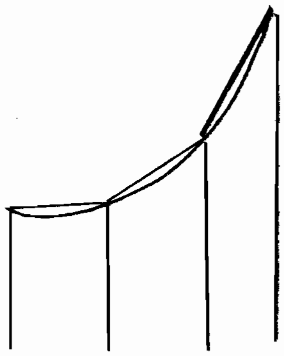
4b)

$$A \approx \frac{h}{2}\left[y_0 + 2(y_1+y_2+y_3+y_4+y_5) + y_6\right]$$

$$A \approx \frac{0.5}{2}\left[1.414 + 2(1.554 + 1.732 + 1.957 + 2.236 + 2.580) + 3\right]$$

$$A \approx 6.133$$

c)



Trapezia extend above curve so approximation is an overestimate

5) 3rd term $ar^2 = 324$ ①

6th term $ar^5 = 96$ ②

a) ② ÷ ①

$$\frac{ar^5}{ar^2} = \frac{96}{324}$$

$$r^3 = \frac{8}{27}$$

$$r = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$$

b)

$$a \times \left(\frac{2}{3}\right)^2 = 324$$

$$\frac{4}{9}a = 324$$

$$4a = 2916$$

$$a = \frac{2916}{4}$$

$$a = 729$$

c)

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{15} = \frac{729\left(1-\left(\frac{2}{3}\right)^{15}\right)}{1-2/3}$$

$$S_{15} = 3 \times 729\left(1-\left(\frac{2}{3}\right)^{15}\right)$$

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5c) cont

$S_{15} = 2182$ to 4 s.f.

5d)

$S_\infty = \frac{a}{1-r}$

$= \frac{729}{1-\frac{2}{3}}$

$= 729 \times 3$

$= 2187$

6) $x^2 + y^2 - 6x + 4y = 12$

a) $(x-3)^2 - 9 + (y+2)^2 - 4 = 12$

$(x-3)^2 + (y+2)^2 = 12 + 9 + 4$

$(x-3)^2 + (y+2)^2 = 25 = 5^2$

Circle centre $(3, -2)$

radius 5

b) Given that $P(-1,1)$ and $Q(7,-5)$ lie on circle

Distance between P and Q

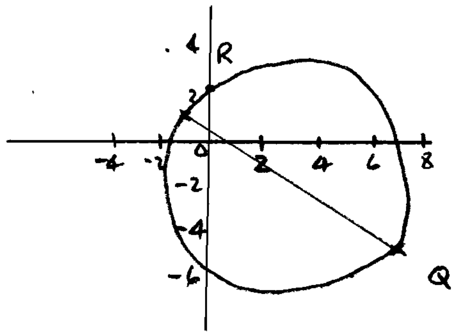
$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(7--1)^2 + (-5-1)^2}$

$= \sqrt{8^2 + (-6)^2}$

$= \sqrt{64+36} = \sqrt{100} = 10$

This is twice the radius so PQ is a diameter



x-coord of R $= 0$

R on circle (since $\angle PRQ = 90°$)

$0^2 + y^2 - 0 + 4y = 12$

$y^2 + 4y - 12 = 0$

$(y+6)(y-2) = 0$

$\Rightarrow y = -6$ or $y = 2$

Told r on positive y-axis

so $R(0, 2)$

7) i) $(1 + \tan\theta)(5\sin\theta - 2) = 0$

Either $1 + \tan\theta = 0$

$\Rightarrow \tan\theta = -1$

or $5\sin\theta - 2 = 0$

$\Rightarrow \sin\theta = \frac{2}{5}$

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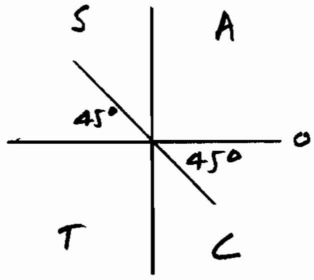
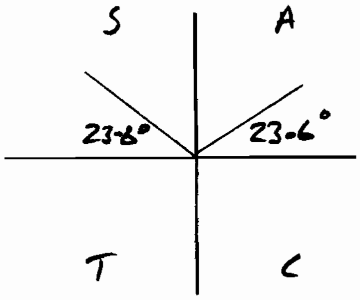
7i) cont)

$\tan^{-1} 1 = 45^\circ$



$\tan\theta = -1$

$\Rightarrow \quad \theta = 135^\circ, -45^\circ$



$\sin^{-1} \frac{2}{5} = 23.6^\circ$

$\sin\theta = \frac{2}{5}$

$\Rightarrow \quad \theta = 23.6^\circ, 156.4^\circ$

Solutions for $-180^\circ \leq \theta < 180^\circ$

$\theta = -45^\circ, 23.6^\circ, 135^\circ, 156.4^\circ$

7ii)

$$4\sin x = 3\tan x$$

$$4\sin x = 3\frac{\sin x}{\cos x}$$

$$4\sin x\cos x = 3\sin x$$

$$4\sin x\cos x - 3\sin x = 0$$

$$\sin x(4\cos x - 3) = 0$$

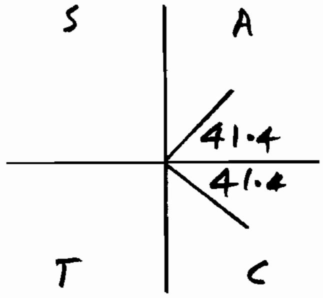
$\Rightarrow \quad \sin x = 0$

or $\quad 4\cos x - 3 = 0$

$\Rightarrow \cos x = \frac{3}{4}$

$\sin x = 0 \quad \Rightarrow \quad x = 0, 180^\circ$

$\cos^{-1}\frac{3}{4} = 41.4^\circ$



$\Rightarrow \quad x = 41.4^\circ, 318.6^\circ$

Solutions for $0^\circ \leq x < 360^\circ$

$x = 0^\circ, 41.4^\circ, 180^\circ, 318.6^\circ$

8a)

$$\log_2 y = -3$$

$$\Rightarrow \quad y = 2^{-3}$$

$$\Rightarrow \quad y = \frac{1}{2^3}$$

$$\Rightarrow \quad y = \frac{1}{8}$$

8b)

$$\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$$

$$\Rightarrow \quad 5 + 4 = (\log_2 x)^2$$

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8b) cont

$\Rightarrow \quad 9 = (\log_2 x)^2$

$\Rightarrow \quad \log_2 x = \pm\sqrt{9}$

$\Rightarrow \quad \log_2 x = \pm 3$

$\Rightarrow \quad x = 2^3 \quad$ or $\quad x = 2^{-3}$

$\Rightarrow \quad x = 8 \quad$ or $\quad x = \frac{1}{8}$

9) a) Top area $= \frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times 1$

$= \frac{1}{2}r^2$

Bottom area

Also $\frac{1}{2}r^2\theta$

Side Rectangle $= rh$

Other side rectangle $= rh$ also

Curved surface $=$ Arc $\times h$
$= r\theta h$
$= r \times 1 \times h$
$= rh$

Total surface area

$= \frac{1}{2}r^2 + \frac{1}{2}r^2 + rh + rh + rh$

$= r^2 + 3rh$

Now volume

$=$ Area of Sector $\times h$

$= \frac{1}{2}r^2\theta h = \frac{1}{2}r^2 h$

$\therefore \quad \frac{1}{2}r^2 h = 300 \text{ cm}^3$

$\Rightarrow \quad h = \frac{600}{r^2}$

Subst for h in total surface area

Surface area $= r^2 + 3r\left(\frac{600}{r^2}\right)$

$= r^2 + \frac{1800}{r}$

9b) $\quad S = r^2 + 1800r^{-1}$

$\frac{dS}{dr} = 2r - 1800r^{-2}$

$\frac{dS}{dr} = 2r - \frac{1800}{r^2}$

S stationary when $\frac{dS}{dr} = 0$

$\Rightarrow \quad 2r - \frac{1800}{r^2} = 0$

$\Rightarrow \quad 2r^3 - 1800 = 0$

$2r^3 = 1800$

$r^3 = 900$

$r = \sqrt[3]{900}$

$r = 9.655$ to 4 s.f.

9c) $\quad \frac{d^2S}{dr^2} = 2 + 3600r^{-3}$

$= 2 + \frac{3600}{r^3}$

> 0 when $r = 9.655$

$\therefore$ a minimum value for S

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9d) Min S

$$S = r^2 + \frac{1800}{r}$$

$$= 9.655^2 + \frac{1800}{9.655}$$

$$= 279.65$$

$$= 280 \text{ cm}^2$$

to nearest cm^2