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1) a) $f(x) = 2x^3 - 3x^2 - 39x + 20$

$f(-4) = 2(-4)^3 - 3(-4)^2 - 39(-4) + 20$

$= -128 - 48 + 156 + 20$

$= -176 + 176 = 0$

$\therefore$ by factor theorem

$(x+4)$ is a factor of $f(x)$

b)

$$\begin{array}{r|l} & 2x^2 - 11x + 5 \\ \hline x+4 & 2x^3 - 3x^2 - 39x + 20 \\ & \underline{2x^3 + 8x^2} \\ & \quad -11x^2 - 39x \\ & \quad \underline{-11x^2 - 44x} \\ & \qquad\quad +5x + 20 \\ & \qquad\quad \underline{+5x + 20} \end{array}$$

$f(x) = (x+4)(2x^2 - 11x + 5)$

$f(x) = (x+4)(2x-1)(x-5)$

2a) $y = \sqrt{(5^x + 2)}$

x	y
0	1.732
0.5	2.058
1	2.646
1.5	3.630
2	5.196

2b) Trapezium Rule

$A = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3) + y_4\right]$

$= \frac{0.5}{2}\left[1.732 + 2(2.058 + 2.646 + 3.630) + 5.196\right]$

$A = 5.899$

$\int_0^2 \sqrt{(5^x + 2)}\,dx \approx 5.899$

3a) $(1 + ax)^{10}$

$= 1 + \binom{10}{1}(ax) + \binom{10}{2}(ax)^2 + \binom{10}{3}(ax)^3 + \cdots$

$= 1 + 10ax + 45a^2x^2 + 120a^3x^3 + \cdots$

3b) Given $120a^3 = 2(45a^2)$

$\Rightarrow 120a^3 = 90a^2$

$\Rightarrow 120a = 90$

$a = \frac{90}{120}$

$a = \frac{3}{4}$

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4a)

$$5^x = 7$$

$$\Rightarrow \log_{10}(5^x) = \log_{10} 7$$

$$\Rightarrow x \log_{10} 5 = \log_{10} 7$$

$$\Rightarrow x = \frac{\log_{10} 7}{\log_{10} 5}$$

$$x = 1.209$$

$$x = 1.21 \text{ to 3 s.f.}$$

4b)

$$5^{2x} - 12(5^x) + 35 = 0$$

$$(5^x - 5)(5^x - 7) = 0$$

Either $5^x - 7 = 0$

$$\Rightarrow 5^x = 7$$

$$\Rightarrow \underline{x = 1.21} \text{ to 3 s.f.}$$

or $5^x - 5 = 0$

$$\Rightarrow 5^x = 5$$

$$\Rightarrow \underline{x = 1}$$

5a) Circle C has centre $(3, 1)$

passes through $P(8, 3)$

$$\text{Radius} = \sqrt{(8-3)^2 + (3-1)^2}$$

$$= \sqrt{25 + 4} = \sqrt{29}$$

Eqn for C

$$(x-3)^2 + (y-1)^2 = 29$$

5b) Tangent at P is $\perp$ to radius at P

Gradient of radius at P is given by $\frac{3-1}{8-3} = \frac{2}{5}$

$\therefore$ gradient of tgt at $P = -\frac{5}{2}$

Using $y - y_1 = m(x - x_1)$

$$y - 3 = -\frac{5}{2}(x - 8)$$

$$\Rightarrow 2y - 6 = -5(x - 8)$$

$$\Rightarrow 2y - 6 = -5x + 40$$

$$\Rightarrow 5x + 2y - 6 - 40 = 0$$

$$\Rightarrow 5x + 2y - 46 = 0$$

6a) Geometric Series

$$a = 5, \quad r = \frac{4}{5}$$

$$20^{th} \text{ term} = ar^{19}$$

$$= 5 \times 0.8^{19}$$

$$= 0.072 \text{ to 3 d.p.}$$

6b)

$$S_\infty = \frac{a}{1-r} = \frac{5}{1-0.8} = 25$$

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6c) $S_k = \dfrac{a(1-r^k)}{1-r}$

$S_k = \dfrac{5(1-0.8^k)}{1-0.8}$

$S_k = 25(1-0.8^k)$

Given $S_k > 24.95$

$\therefore\ 25(1-0.8^k) > 24.95$

$\Rightarrow\ 1-0.8^k > \dfrac{24.95}{25}$

$1-0.8^k > 0.998$

$1-0.998 > 0.8^k$

$0.002 > 0.8^k$

$\Rightarrow\ \log 0.002 > \log(0.8)^k$

$\Rightarrow\ \log 0.002 > k\log(0.8)$

$\Rightarrow\ \dfrac{\log 0.002}{\log 0.8} < k$

(since log 0.8 is negative inequality is reversed when divided through by a negative)

so $k > \dfrac{\log 0.002}{\log 0.8}$

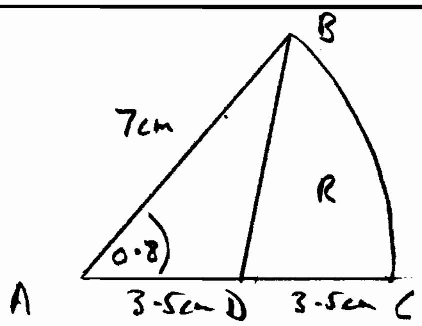
as required

6d) $k > \dfrac{\log_{10} 0.002}{\log_{10} 0.8} = 27.85$

$\therefore$ smallest value of k is

$k = 28$

7.



a) Arc length $= r\theta$

$= 7 \times 0.8 = 5.6$ cm

b) Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 7^2 \times 0.8$

$= 19.6$ cm^2

c) First find $|BD|$ using cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$

$BD^2 = 7^2 + 3.5^2 - 2 \times 7 \times 3.5\cos 0.8$

$BD^2 = 27.111$

$BD = \sqrt{27.111} = 5.207$ cm

Permeter $= BD + DC + \text{Arc } BC$

$= 5.21 + 3.5 + 5.6$

$= 14.31$ cm

$= 14.3$ cm to 3 s.f

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7d) Area of R = Area of sector − Area of Δ

Find area of Δ using

$$\text{Area} = \frac{1}{2}ab\sin C$$

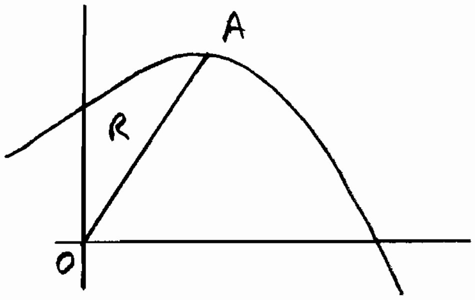
$$\text{Area of } \Delta = \frac{1}{2} \times 7 \times 3.5 \sin 0.8$$

$$= 8.79 \text{ cm}^2$$

$$\text{Area of } R = 19.6 - 8.79$$

$$= 10.8 \text{ cm}^2 \text{ to } 3 \text{ s.f.}$$

8a)



$$y = 10 + 8x + x^2 - x^3$$

$$\frac{dy}{dx} = 8 + 2x - 3x^2$$

When $x = 2$, $\frac{dy}{dx} = 8 + 2(2) - 3(2)^2$

$$= 8 + 4 - 12 = 0$$

$\therefore$ gradient $= 0$ when $x = 2$

so A has x-coord $= 2$

8b) When $x = 2$,

$$y = 10 + 8(2) + 2^2 - 2^3$$

$$= 10 + 16 + 4 - 8 = 22$$

A is point $(2, 22)$

$\therefore$ OA is line $y = \frac{22}{2}x$

$$y = 11x$$

Area R is given by

$$\int_0^2 (y_{curve} - y_{line})\, dx$$

$$= \int_0^2 (10 + 8x + x^2 - x^3 - 11x)\, dx$$

$$= \int_0^2 (10 - 3x + x^2 - x^3)\, dx$$

$$= \left[10x - \frac{3x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \right]_0^2$$

$$= \left(10(2) - \frac{3(2)^2}{2} + \frac{2^3}{3} - \frac{2^4}{4} \right) - (0)$$

$$= 20 - 6 + \frac{8}{3} - 4$$

$$= 10 + \frac{8}{3} = \frac{38}{3} \text{ units}^2$$

9a) for $0 \leq x < 360$

Solve $\sin(x - 20°) = \frac{1}{\sqrt{2}}$

$$\sin^{-1}\frac{1}{\sqrt{2}} = 45°$$

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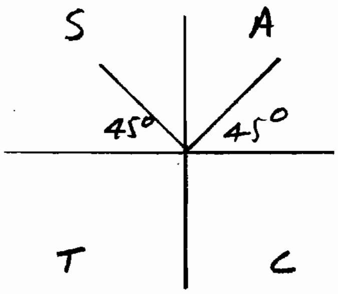
9a cont



$\therefore x - 20° = 45°, 135°$

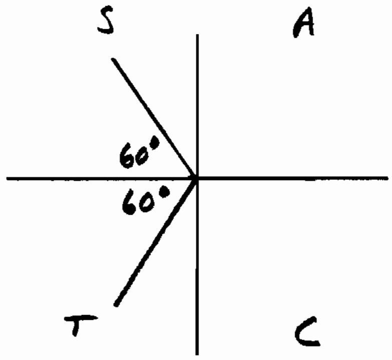
$\Rightarrow x = 65°$ or $155°$

9b) Solve $\cos 3x = -\frac{1}{2}$

Need to consider

$0 \le 3x \le 3 \times 360°$

$\cos^{-1}\frac{1}{2} = 60°$



Cosine negative in 2nd and 3rd quadrants

$3x = 120°, 240°$

$480°, 600°$

$840°, 960°$

$\therefore x = 40°, 80°, 160°, 200°$
$280°, 320°$