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1a) Let $f(x) = x^3 - 2x^2 - 4x + 8$

i) Remainder when $f(x)$ is divided by $(x-3)$ is given by $f(3)$

$$f(3) = 3^3 - 2(3)^2 - 4(3) + 8$$
$$= 27 - 18 - 12 + 8$$
$$= 5$$

ii) Remainder when dividing by $(x+2)$ is given by $f(-2)$

$$f(-2) = (-2)^3 - 2(-2)^2 - 4(-2) + 8$$
$$= -8 - 8 + 8 + 8$$
$$= 0$$

This result means that $(x+2)$ is actually a factor of $f(x)$

b) Use algebraic long division to find other quadratic factor

$$\begin{array}{r|l} & x^2 - 4x + 4 \\ \hline x+2 & x^3 - 2x^2 - 4x + 8 \\ & x^3 + 2x^2 \\ \hline & \quad -4x^2 - 4x \\ & \quad -4x^2 - 8x \\ \hline & \qquad +4x + 8 \\ & \qquad +4x + 8 \\ \hline \end{array}$$

So $f(x) = (x+2)(x^2 - 4x + 4)$
$= (x+2)(x-2)(x-2)$

Solve $x^3 - 2x^2 - 4x + 8 = 0$
$(x+2)(x-2)(x-2) = 0$

Either $x = -2$

or $x = +2$

Geometric Series

2.a) 4^{th} term $ar^3 = 10$ ①

7^{th} term $ar^6 = 80$ ②

② ÷ ①

$$\frac{ar^6}{ar^3} = \frac{80}{10}$$

$\Rightarrow \quad r^3 = 8$

$\Rightarrow \quad r = 2$

Sub for r in ①

$a \times 2^3 = 10 \quad \Rightarrow 8a = 10$

2b) $\Rightarrow a = 1.25$

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2c) $$S_n = \frac{a(r^n - 1)}{r-1}$$

$$S_{20} = \frac{1.25(2^{20}-1)}{2-1}$$

$$S_{20} = 1,310,718.75$$

$$= 1,310,719$$

to nearest whole number

3) a) $$\left(1+\frac{x}{2}\right)^{10}$$

First 4 terms

$$1 + \binom{10}{1}\left(\frac{x}{2}\right) + \binom{10}{2}\left(\frac{x}{2}\right)^2 + \binom{10}{3}\left(\frac{x}{2}\right)^3$$

$$= 1 + 5x + 11.25x^2 + 15x^3$$

3b) $$(1.005)^{10} = \left(1+\frac{0.01}{2}\right)^{10}$$

$$\approx 1 + 5(0.01) + 11.25(0.01)^2 + 15(0.01)^3$$

$$\approx 1.05114$$

4) Using $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$$

$\therefore$ eqn $3\sin^2\theta - 2\cos^2\theta = 1$

can be written as

$$3\sin^2\theta - 2(1-\sin^2\theta) = 1$$

$$3\sin^2\theta - 2 + 2\sin^2\theta = 1$$

$$5\sin^2\theta = 1 + 2$$

$$5\sin^2\theta = 3$$

b) for $0 \le \theta < 360°$

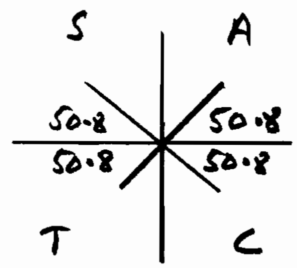
Solve $3\sin^2\theta - 2\cos^2\theta = 1$

$$\Rightarrow 5\sin^2\theta = 3$$

$$\Rightarrow \sin^2\theta = \frac{3}{5}$$

$$\Rightarrow \sin\theta = \pm\sqrt{\frac{3}{5}}$$

$$\sin^{-1}\sqrt{\frac{3}{5}} = 50.8°$$



$\theta = 50.8°$

$\theta = 180° - 50.8° = 129.2°$

$\theta = 180° + 50.8° = 230.8°$

$\theta = 360° - 50.8° = 309.2°$

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5)

$$a = 3b \quad (1)$$

$$\log_3 a + \log_3 b = 2 \quad (2)$$

From (2) $\log_3(ab) = 2 \quad (3)$

Subst for a in (3)

$$\log_3(3b^2) = 2$$

$$\log_3 3 + \log_3 b^2 = 2$$

$$1 + 2\log_3 b = 2$$

$$2\log_3 b = 2 - 1$$

$$2\log_3 b = 1$$

$$\log_3 b = \tfrac{1}{2}$$

$$\Rightarrow b = 3^{\frac{1}{2}} = \sqrt{3}$$

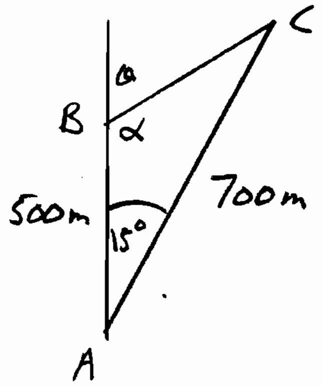
Subst for b in (1)

$$a = 3\sqrt{3}$$

Solution $a = 3\sqrt{3}$

$b = \sqrt{3}$

6 a)



cosine rule

$$BC^2 = 500^2 + 700^2 - 2 \times 500 \times 700 \cos 15^\circ$$

$$BC = \sqrt{500^2 + 700^2 - 2 \times 500 \times 700 \cos 15^\circ}$$

$$BC = 252.69 \text{ m}$$

$BC = 253$ m to 3 sig fig

b) Find α first

sine rule

$$\frac{700}{\sin\alpha} = \frac{252.69}{\sin 15^\circ}$$

$$700 \sin 15^\circ = 252.69 \sin\alpha$$

$$\sin\alpha = \frac{700 \sin 15^\circ}{252.69}$$

$$\alpha = \sin^{-1}\left(\frac{700 \sin 15^\circ}{252.69}\right)$$

$$\alpha = 45.8^\circ$$

$$\Rightarrow \theta = 180 - \alpha$$

$$= 180 - 45.8^\circ$$

$$\theta = 134.2^\circ$$

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7.

a) $C: \quad y = 6x - x^2$

When $y = 0$, $\quad 0 = 6x - x^2$

$$0 = x(6 - x)$$

$$\Rightarrow \quad x = 0 \text{ or } x = 6$$

$\therefore$ intersects x-axis at

$(0,0)$ and $(6,0)$

b) $L: \quad y = 2x$

Solve $\begin{cases} y = 2x & (1) \\ y = 6x - x^2 & (2) \end{cases}$

Sub for y in (2)

$$2x = 6x - x^2$$

$$0 = 4x - x^2$$

$$0 = x(4 - x)$$

$$\Rightarrow \quad x = 0 \text{ or } x = 4$$

When $x = 0$
$y = 2 \times 0 = 0$

when $x = 4$
$y = 2 \times 4 = 8$

$\therefore$ L and C intersect at

$(0,0)$ and $(4,8)$

7c) $$R = \int_0^4 (y_c - y_L)\,dx$$

$$R = \int_0^4 (6x - x^2 - 2x)\,dx$$

$$= \int_0^4 (4x - x^2)\,dx$$

$$= \left[2x^2 - \frac{x^3}{3} \right]_0^4$$

$$= \left(2 \times 4^2 - \frac{4^3}{3}\right) - (0 - 0)$$

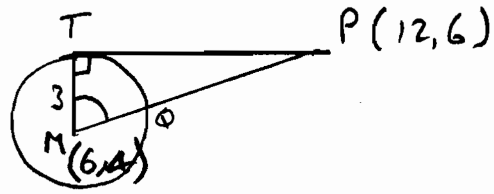
$$= 32 - \frac{64}{3}$$

$$= \frac{96}{3} - \frac{64}{3} = \frac{32}{3} \text{ units}^2$$

8) a) Centre $(6,4)$ radius 3

$$(x-6)^2 + (y-4)^2 = 3^2$$

b)



Find $|PM| = \sqrt{(12-6)^2 + (6-4)^2}$

$$= \sqrt{36 + 4} = \sqrt{40}$$

$\angle MTP = 90°$ so

$$\cos(\angle TMQ) = \frac{3}{\sqrt{40}}$$

$\angle TMQ = 1.07658$ radians $= 1.0766$ to 4 d.p.

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8c) Shaded Area =

Area of ΔPTM − Area of Sector MTQ

Area of $\Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 3 \times \sqrt{40} \times \sin 1.0766$

$= 8.3517$

Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 3^2 \times 1.0766$

$= 4.8447$

Shaded Area

$= 8.3517 - 4.8447$

$= 3.507$ units^2

9.) $A = x^2 + x^2 + xy + xy + xy$

a) $A = 2x^2 + 3xy$

Now capacity $x^2y = 100$

$\Rightarrow y = \frac{100}{x^2}$

$\therefore A = 2x^2 + 3x \times \frac{100}{x^2}$

$A = 2x^2 + \frac{300}{x}$

b) $A = 2x^2 + 300x^{-1}$

$\frac{dA}{dx} = 4x - 300x^{-2}$

$= 4x - \frac{300}{x^2}$

Stationary when $\frac{dA}{dx} = 0$

$\Rightarrow 4x - \frac{300}{x^2} = 0$

$\Rightarrow 4x^3 - 300 = 0$

$\Rightarrow 4x^3 = 300$

$\Rightarrow x^3 = 75$

$\Rightarrow x = 4.217$

c) $\frac{d^2A}{dx^2} = 4 + 600x^{-3}$

$= 4 + \frac{600}{x^3}$

when $x = 4.217$, $\frac{d^2A}{dx^2} > 0$

$\therefore$ a min value of A

d) Min $A = 2(4.217)^2 + \frac{300}{4.217}$

$= 106.71 \text{ m}^2$

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