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1) $\int (2 + 5x^2)\,dx$

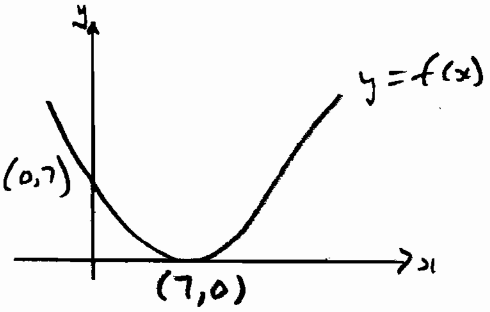
$= 2x + \dfrac{5x^3}{3} + c$

2) $x^3 - 9x$

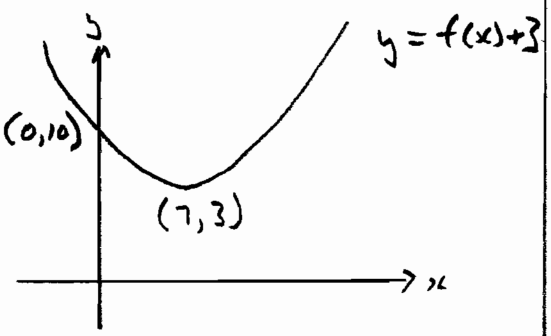
$= x(x^2 - 9)$

$= x(x+3)(x-3)$

3)

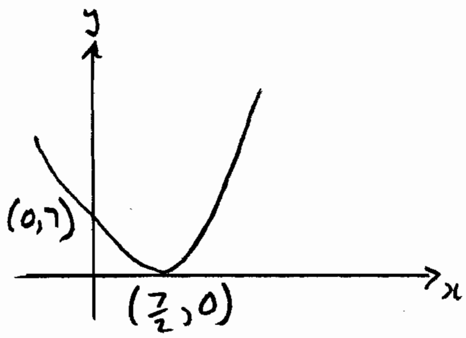


a)



(A translation by $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$)

b)



(One way stretch parallel to x-axis scale factor $\frac{1}{2}$)

4) $f(x) = 3x + x^3$, $x > 0$

a) $f'(x) = 3 + 3x^2$

b) Given $f'(x) = 15$

$\Rightarrow 3 + 3x^2 = 15$

$\Rightarrow 3x^2 = 15 - 3$

$\Rightarrow 3x^2 = 12$

$\Rightarrow x^2 = 4$

$\Rightarrow x = +2$

or $\cancel{x = -2}$ since $x > 0$

5) $x_1 = 1$

$x_{n+1} = ax_n - 3$, $n \geq 1$

a) $x_2 = ax_1 - 3$

$x_2 = a \times 1 - 3$

$x_2 = a - 3$

b) $x_3 = ax_2 - 3$

$x_3 = a(a-3) - 3$

$x_3 = a^2 - 3a - 3$

c) If $x_3 = 7$

$a^2 - 3a - 3 = 7$

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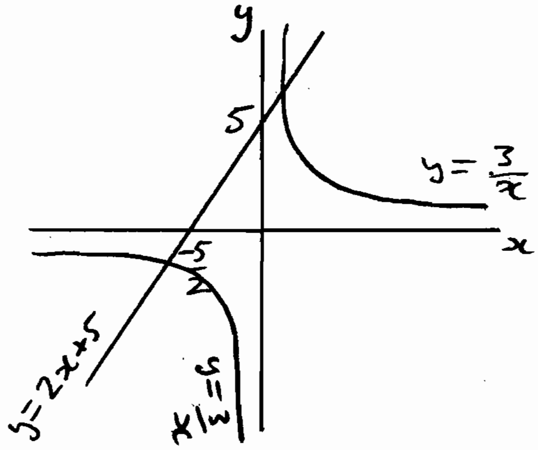
5c cont)

$\Rightarrow a^2 - 3a - 10 = 0$

$\Rightarrow (a-5)(a+2) = 0$

$\Rightarrow a = 5$ or $a = -2$

6) a)



b)

$y = \frac{3}{x}$ ①

$y = 2x + 5$ ②

Sub for y in ②

$\frac{3}{x} = 2x + 5$

$\Rightarrow 3 = 2x^2 + 5x$

$\Rightarrow 2x^2 + 5x - 3 = 0$

$\Rightarrow (2x-1)(x+3) = 0$

$\Rightarrow 2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

or $x + 3 = 0$

$x = -3$

When $x = \frac{1}{2}$ $y = 2(\frac{1}{2}) + 5 = 6$

when $x = -3$ $y = 2(-3) + 5 = -1$

$\therefore$ points of intersection are

$(\frac{1}{2}, 6)$ and $(-3, -1)$

7) a)

1st Sat	5 km
2nd Sat	7 km
3rd Sat	9 km
4th Sat	11 km

b) AP with $a = 5$, $d = 2$

n^{th} SAT $a + (n-1)d$

$= 5 + 2(n-1)$

$= 5 + 2n - 2$

$= 3 + 2n$ km

c) $S_n = \frac{n}{2}(2a + (n-1)d)$

$\Rightarrow S_n = \frac{n}{2}(2 \times 5 + 2(n-1))$

$S_n = \frac{n}{2}(8 + 2n)$

$S_n = 4n + n^2$

$S_n = n(4+n)$ km

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7d) n^{th} SAT $= 3 + 2n$ km

$\Rightarrow 3 + 2n = 43$

$2n = 43 - 3$

$2n = 40$

$n = 20$

7e) Total distance

$= n(4+n)$ km

$= 20(4+20)$ km

$= 480$ km

8a) $2qx^2 + qx - 1 = 0$

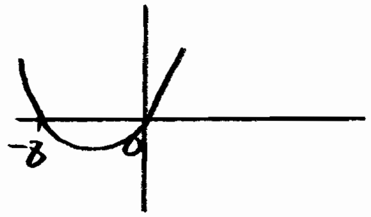
If no real roots, discriminant $b^2 - 4ac < 0$

$\Rightarrow \quad q^2 - 4(2q)(-1) < 0$

$q^2 + 8q < 0$

8b) $q(q+8) < 0$

Graph of $y = q^2 + 8q$



$-8 < q < 0$

9a) $y = kx^3 - x^2 + x - 5$

$\dfrac{dy}{dx} = 3kx^2 - 2x + 1$

9b) Line $2y - 7x + 1 = 0$

$2y = 7x - 1$

$y = \frac{7}{2}x - \frac{1}{2}$

This line has gradient $\frac{7}{2}$

Tangent at $x = -\frac{1}{2}$ is parallel so also has gradient $\frac{7}{2}$

$\therefore \dfrac{dy}{dx} = \dfrac{7}{2}$ at $x = -\frac{1}{2}$

$\Rightarrow 3k(-\frac{1}{2})^2 - 2(-\frac{1}{2}) + 1 = \frac{7}{2}$

$\frac{3k}{4} + 1 + 1 = \frac{7}{2}$

$3k + 4 + 4 = 14$

$3k = 14 - 4 - 4$

$3k = 6$

$k = 2$

9c) $x = -\frac{1}{2}$ at A

$y = 2(-\frac{1}{2})^3 - (-\frac{1}{2})^2 + (-\frac{1}{2}) - 5$

$y = -\frac{2}{8} - \frac{1}{4} - \frac{1}{2} - 5$

$y = -6$

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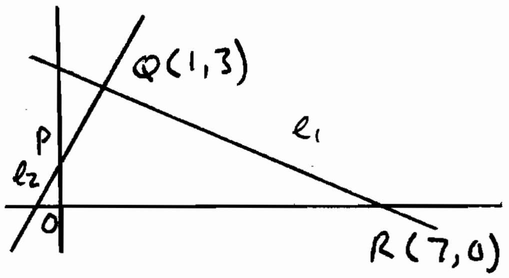
10)
a)



$|QR| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(7-1)^2+(0-3)^2}$

$= \sqrt{36+9}$

$= \sqrt{45}$

$= \sqrt{9\times 5}$

$= 3\sqrt{5}$

$\therefore a = 3$

b) Gradient o $l_1 = \dfrac{y_2-y_1}{x_2-x_1}$

$= \dfrac{0-3}{7-1}$

$= -\dfrac{3}{6} = -\dfrac{1}{2}$

$l_2 \perp$ to l_1

so gradient of $l_2 = 2$

Passes through Q (1,3)

Use $y-y_1 = m(x-x_1)$

$y-3 = 2(x-1)$

$y-3 = 2x-2$

$y = 2x+1$ is l_2

c) At P $x=0$

$y = 2(0)+1 = 1$

P is point $(0,1)$

d) Find $|QP|$

$= \sqrt{(1-0)^2+(3-1)^2}$

$= \sqrt{5}$

Δ has $90°$ angle at Q

Area $= \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} \times |QP| \times |QR|$

$= \frac{1}{2} \times \sqrt{5} \times 3\sqrt{5}$

$= \frac{1}{2} \times 3 \times 5$

$= \dfrac{15}{2}$ units^2

11)
a) $\dfrac{dy}{dx} = \dfrac{(x^2+3)^2}{x^2}$

$\dfrac{dy}{dx} = \dfrac{(x^4+6x^2+9)}{x^2}$

$\dfrac{dy}{dx} = \dfrac{x^4}{x^2} + \dfrac{6x^2}{x^2} + \dfrac{9}{x^2}$

$\dfrac{dy}{dx} = x^2 + 6 + 9x^{-2}$

11b) $(3, 20)$ lies on C

$$\frac{dy}{dx} = x^2 + 6 + 9x^{-2}$$

$$\Rightarrow y = \frac{x^3}{3} + 6x + \frac{9x^{-1}}{-1} + c$$

$$y = \frac{x^3}{3} + 6x - \frac{9}{x} + c$$

Sub $(3,20)$ in curve

$$20 = \frac{3^3}{3} + 6(3) - \frac{9}{3} + c$$

$$20 = 9 + 18 - 3 + c$$

$$20 = 24 + c$$

$$20 - 24 = c$$

$$-4 = c$$

$$\therefore\ y = \frac{x^3}{3} + 6x - \frac{9}{x} - 4$$

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