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1) a) $125^{\frac{1}{3}} = \sqrt[3]{125}$

$= 5$

b) $125^{-2/3} = \dfrac{1}{125^{2/3}}$

$= \dfrac{1}{(\sqrt[3]{125})^2}$

$= \dfrac{1}{5^2}$

$= \dfrac{1}{25}$

2) $\int (12x^5 - 8x^3 + 3)\,dx$

$= \dfrac{12x^6}{6} - \dfrac{8x^4}{4} + 3x + c$

$= 2x^6 - 2x^4 + 3x + c$

3) $(\sqrt{7}+2)(\sqrt{7}-2)$

$= (\sqrt{7})^2 - 2^2$

$= 7 - 4$

$= 3$

That solution was based upon the factorisation of the difference of two squares $a^2 - b^2 = (a+b)(a-b)$

Alternatively, you could just multiply out the brackets

$(\sqrt{7}+2)(\sqrt{7}-2)$

$= 7 + 2\sqrt{7} - 2\sqrt{7} - 4$

$= 3$

4) $y = f(x)$ passes through $(4, 22)$

$f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7$

$\Rightarrow f(x) = \dfrac{3x^3}{3} - \dfrac{3x^{3/2}}{3/2} - 7x + c$

$\Rightarrow y = x^3 - \frac{2}{3} \times 3x^{3/2} - 7x + c$

$\Rightarrow y = x^3 - 2x^{3/2} - 7x + c$

$\Rightarrow 22 = 4^3 - 2 \times 4^{3/2} - 7(4) + c$

$\Rightarrow 22 = 64 - 16 - 28 + c$

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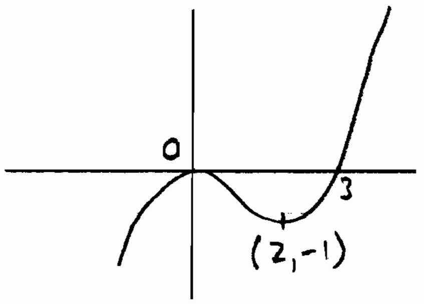
4 cont) $\Rightarrow \quad 22 = 20 + c$

$$22 - 20 = c$$

$$2 = c$$

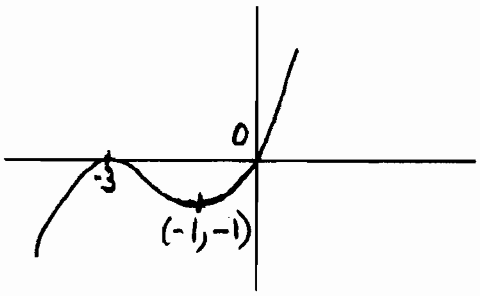
$$\therefore f(x) = x^3 - 2x^{3/2} - 7x + 2$$

5) $y = f(x)$

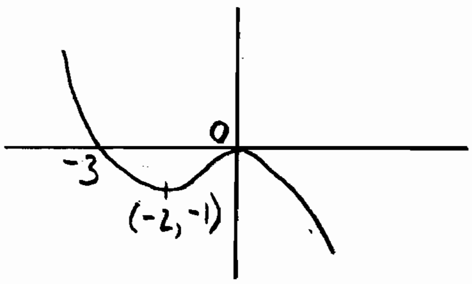


a) $y = f(x+3)$

translation by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$



b) $y = f(-x)$ reflection in y-axis



6) Write $\dfrac{2x^2 - x^{3/2}}{\sqrt{x}}$ as $2x^p - x^q$

a)
$$\frac{2x^2 - x^{3/2}}{\sqrt{x}} = \frac{2x^2 - x^{3/2}}{x^{1/2}}$$

$$= \frac{2x^2}{x^{1/2}} - \frac{x^{3/2}}{x^{1/2}}$$

$$= 2x^{3/2} - x^1$$

$$p = 3/2, \quad q = 1$$

b)
$$y = 5x^4 - 3 + \frac{2x^2 - x^{3/2}}{\sqrt{x}}$$

$$y = 5x^4 - 3 + 2x^{3/2} - x$$

$$\frac{dy}{dx} = 20x^3 + 0 + \frac{3}{2} \times 2x^{\frac{1}{2}} - 1$$

$$\frac{dy}{dx} = 20x^3 + 3x^{\frac{1}{2}} - 1$$

7) $kx^2 + 4x + (5-k) = 0$

a) 2 different real solutions

$$\Rightarrow \quad b^2 - 4ac > 0$$

$$\Rightarrow \quad 4^2 - 4k(5-k) > 0$$

$$\Rightarrow \quad 16 - 20k + 4k^2 > 0$$

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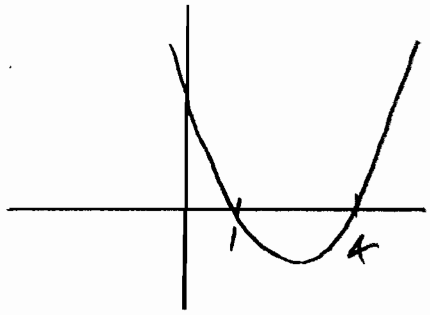
7a) cont) $\Rightarrow 4 - 5k + k^2 > 0$

$\Rightarrow k^2 - 5k + 4 > 0$

b) $k^2 - 5k + 4 > 0$

$(k-4)(k-1) > 0$

Graph of $y = k^2 - 5k + 4$



Either $k > 4$

or $k < 1$

8) $P(1, a)$ on curve

$y = (x+1)^2(2-x)$

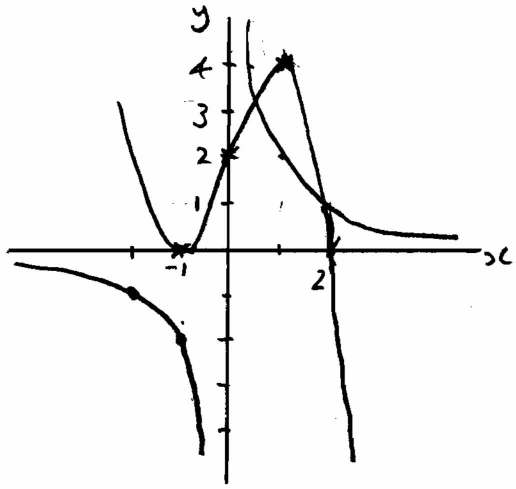
a) when $x = 1$

$y = (1+1)^2(2-1)$

$= 4$

$\therefore \quad a = 4$

b)



c) 2 points of intersection

$\Rightarrow$ 2 real solutions

9) Arithmetic Progression

a) 18^{th} term $a + 17d = 25$ ①

21^{st} term $a + 20d = 32\frac{1}{2}$ ②

b) ② − ① $3d = 7\frac{1}{2}$

$\Rightarrow d = 2\frac{1}{2}$

Subst in ①

$a + 17 \times 2\frac{1}{2} = 25$

$a + 42\frac{1}{2} = 25$

$a = 25 - 42\frac{1}{2}$

$\Rightarrow a = -17\frac{1}{2}$

c) $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

If $S_n = 2750$

$\frac{n}{2}\left(2a + (n-1)d\right) = 2750$

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9c) cont

$\Rightarrow \frac{n}{2}\left(-35 + 2\cdot5(n-1)\right) = 2750$

$\Rightarrow n\left(-35 + 2\cdot5n - 2\cdot5\right) = 5500$

$\Rightarrow n\left(-70 + 5n - 5\right) = 11000$

$\Rightarrow 5n^2 - 75n = 11000$

$\Rightarrow n^2 - 15n = 2200$

$\Rightarrow n^2 - 15n = 55 \times 40$

d) $n^2 - 15n - 55\times40 = 0$

$(n-55)(n+40) = 0$

$\Rightarrow n = 55$

or $n = -40$

In this context $n > 0$

$\therefore n = 55$

10) l_1 through $A(2,5)$ grad $-\frac{1}{2}$

a)

$y - y_1 = m(x - x_1)$

$y - 5 = -\frac{1}{2}(x-2)$

$y - 5 = -\frac{1}{2}x + 1$

$y = -\frac{1}{2}x + 6$

b) $B(-2,7)$

when $x = -2$

$y = -\frac{1}{2}(-2) + 6$

$y = 1 + 6 = 7$

$\therefore B(-2,7)$ is on l_1

c) $|AB| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(-2-2)^2 + (7-5)^2}$

$= \sqrt{(-4)^2 + 2^2}$

$= \sqrt{16 + 4}$

$= \sqrt{20}$

$= \sqrt{4\times5}$

$= 2\sqrt{5}$

d) C on l_1

so when $x = p$

$y = -\frac{1}{2}p + 6$

$|AC| = \sqrt{(p-2)^2 + \left(-\frac{1}{2}p + 6 - 5\right)^2}$

$\Rightarrow 5 = \sqrt{(p-2)^2 + (1 - \frac{1}{2}p)^2}$

$25 = p^2 - 4p + 4 + 1 - p + \frac{1}{4}p^2$

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10d) cont)

$$25 = \frac{5}{4}p^2 - 5p + 5$$

$$5 = \frac{1}{4}p^2 - p + 1$$

$$20 = p^2 - 4p + 4$$

$$0 = p^2 - 4p + 4 - 20$$

$$0 = p^2 - 4p - 16$$

11) $y = 9 - 4x - \frac{8}{x}$, $x > 0$

a) First find $\frac{dy}{dx}$

$$y = 9 - 4x - 8x^{-1}$$

$$\frac{dy}{dx} = -4 - 1(-8)x^{-2}$$

$$= -4 + \frac{8}{x^2}$$

When $x = 2$ at P

$$\frac{dy}{dx} = -4 + \frac{8}{2^2}$$

$$= -4 + 2$$

$$= -2$$

This is also gradient of tangent at P

Find y coord at P

When $x = 2$

$$y = 9 - 4(2) - \frac{8}{2}$$

$$= 9 - 8 - 4 \qquad = -3$$

$\therefore$ P is point $(2, -3)$

tangent goes through $(2, -3)$ with gradient -2

$$y - y_1 = m(x - x_1)$$

$$y - -3 = -2(x - 2)$$

$$y + 3 = -2x + 4$$

$$y = -2x + 4 - 3$$

$$y = -2x + 1$$

so $y = 1 - 2x$ as required

b) Normal has gradient $+\frac{1}{2}$ since perpendicular gradients multiply to -1

$$y - y_1 = m(x - x_1)$$

$$y - -3 = \frac{1}{2}(x - 2)$$

$$y + 3 = \frac{1}{2}x - 1$$

$$y = \frac{1}{2}x - 1 - 3$$

$$y = \frac{1}{2}x - 4$$

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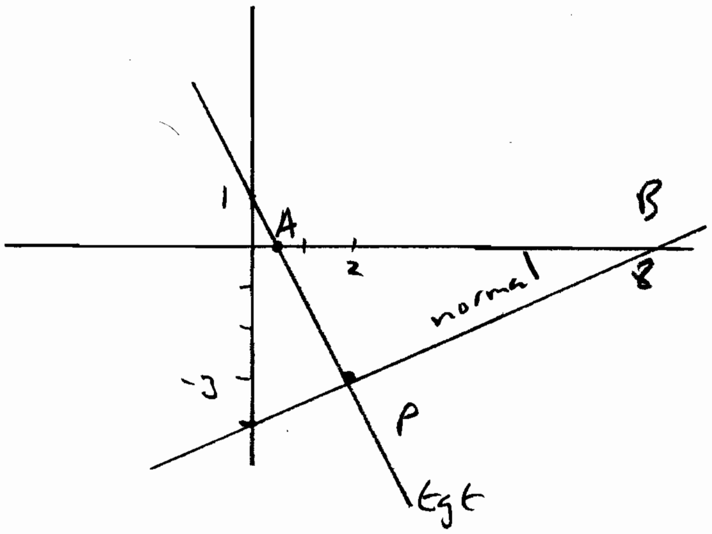
11c)



For tangent

Find where meets x-axis

$\Rightarrow y = 0$

$$y = 1 - 2x$$
$$0 = 1 - 2x$$
$$2x = 1$$
$$x = \tfrac{1}{2}$$

so $A\left(\tfrac{1}{2}, 0\right)$

Find where normal meets x-axis

$\Rightarrow y = 0$

$$y = \tfrac{1}{2}x - 4$$
$$0 = \tfrac{1}{2}x - 4$$
$$4 = \tfrac{1}{2}x$$
$$8 = x$$

so $B(8, 0)$

Area of $\Delta = \tfrac{1}{2}$ base $\times$ height

Take base $AB = 8 - \tfrac{1}{2} = \tfrac{15}{2}$

Height is vertical distance of P below x-axis $= 3$ units

$$\text{Area} = \frac{1}{2} \times \frac{15}{2} \times 3$$

$$= \frac{45}{4} \text{ units}^2$$

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