EDEXCEL CORE 1 JAN 2008

1)

$$\int (3x^2 + 4x^5 - 7)\,dx$$

$$= \frac{3x^3}{3} + \frac{4x^6}{6} - 7x + c$$

$$= x^3 + \frac{2x^6}{3} - 7x + c$$

2) a) $16^{\frac{1}{4}} = \sqrt[4]{16} = 2$

b) $(16x^{12})^{\frac{3}{4}}$

$$= \left(\sqrt[4]{16x^{12}}\right)^3$$

$$= (2x^3)^3$$

$$= 8x^9$$

3)

$$\frac{5 - \sqrt{3}}{2 + \sqrt{3}}$$

$$= \frac{5 - \sqrt{3}}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$= \frac{10 - 2\sqrt{3} - 5\sqrt{3} + 3}{2^2 - \sqrt{3}^2}$$

$$= \frac{13 - 7\sqrt{3}}{1}$$

$$= 13 - 7\sqrt{3}$$

Question 4b accidentally omitted. Solution is on Page 5

4) a) $A(-6, 4)$ $B(8, -3)$

Use $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$

$$\frac{y - 4}{-3 - 4} = \frac{x - -6}{8 - -6}$$

$$\Rightarrow \frac{y - 4}{-7} = \frac{x + 6}{14}$$

$$\Rightarrow 14(y - 4) = -7(x + 6)$$

$$\Rightarrow 2(y - 4) = -1(x + 6)$$

$$\Rightarrow 2y - 8 = -x - 6$$

$$\Rightarrow x + 6 + 2y - 8 = 0$$

$$\Rightarrow x + 2y - 2 = 0$$

5) a)

$$\frac{2\sqrt{x} + 3}{x}$$

$$= \frac{2\sqrt{x}}{x} + \frac{3}{x}$$

$$= \frac{2x^{\frac{1}{2}}}{x^1} + 3x^{-1}$$

$$= 2x^{-\frac{1}{2}} + 3x^{-1}$$

b) $y = 5x - 7 + \dfrac{2\sqrt{x} + 3}{x}$

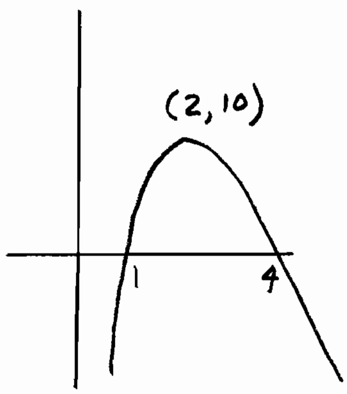
$$\Rightarrow y = 5x - 7 + 2x^{-\frac{1}{2}} + 3x^{-1}$$

EDEXCEL CORE 1 JAN 2008

5b cont) $\frac{dy}{dx} = 5 - x^{-\frac{3}{2}} - 3x^{-2}$

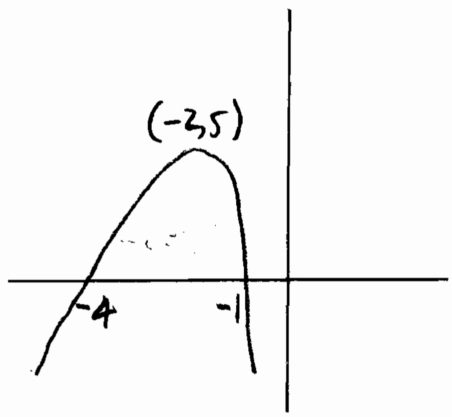
6) a) $y = 2f(x)$

(This represents a one-way stretch parallel to y-axis with scale factor 2)



b) $y = f(-x)$

(This represents a reflection in the y-axis)



c) $y = f(x+a)$ is obtained by a translation of $\begin{pmatrix} -a \\ 0 \end{pmatrix}$

Graph has moved 2 units to left in negative x-direction

so $y = f(x+2)$

Answer $a = 2$

7) $x_1 = 1$, $x_{n+1} = x_n(p + x_n)$

a)

$x_2 = 1(p+1) = p+1$

b)

$x_3 = x_2(p + x_2)$

$= (p+1)(p+p+1)$

$= (p+1)(2p+1)$

$= 2p^2 + 2p + p + 1$

$= 2p^2 + 3p + 1$

c) If $x_3 = 1$

$2p^2 + 3p + 1 = 1$

$2p^2 + 3p = 0$

$p(2p+3) = 0$

$\Rightarrow$ either $p = 0$

or $2p + 3 = 0$

$2p = -3$

$p = -\frac{3}{2}$

Given $p \neq 0$ so $p = -\frac{3}{2}$

EDEXCEL CORE 1 JAN 2008

7d)

$x_3 = 1$

$x_4 = 1(p+1)$

$= 1(-\frac{3}{2}+1)$

$= -\frac{1}{2}$

$x_5 = -\frac{1}{2}\left(-\frac{3}{2} + -\frac{1}{2}\right)$

$= -\frac{1}{2}(-2)$

$= 1$

Sequence is periodic with period 2

$x_{2008} = -\frac{1}{2}$

8)

$x^2 + kx + 8 = k$

$\Rightarrow x^2 + kx + (8-k) = 0$

For no real solutions $b^2 < 4ac$

$\Rightarrow k^2 < 4 \times 1 \times (8-k)$

$\Rightarrow k^2 < 32 - 4k$

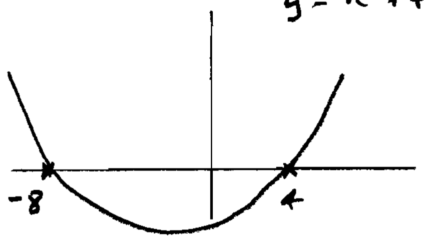
$\Rightarrow k^2 + 4k - 32 < 0$

$(k+8)(k-4) < 0$

Draw graph of

$y = k^2 + 4k - 32$

$y = k^2 + 4k - 32$



Answer $-8 < k < 4$

9)a) $y = f(x)$

$f'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2}$

$f'(x) = 4x - 6x^{\frac{1}{2}} + 8x^{-2}$

$\Rightarrow f(x) = 2x^2 - \frac{6x^{3/2}}{\frac{3}{2}} + \frac{8x^{-1}}{-1} + c$

$\Rightarrow f(x) = 2x^2 - 4x^{3/2} - \frac{8}{x} + c$

$(4, 1)$ on C so

$1 = 2(4)^2 - 4 \times 4^{3/2} - \frac{8}{4} + c$

$1 = 32 - 32 - 2 + c$

$3 = c$

$\therefore f(x) = 2x^2 - 4x^{3/2} - \frac{8}{x} + 3$

9b) Accidentally omitted

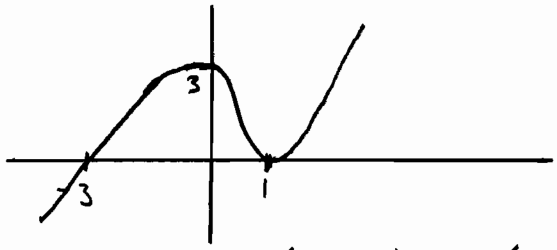
Solution on Page 5

EDEXCEL CORE 1 JAN 2008

10) a)

$$y = (x+3)(x-1)^2$$

y intercept $= (+3)\times(-1)\times(-1) = 3$



Cuts axes at $(-3,0)$ and $(0,3)$
Touches x-axis at $(1,0)$

b)

$$y = (x+3)(x-1)(x-1)$$

$$y = (x+3)(x^2-2x+1)$$

$$y = x^3 + 3x^2 - 2x^2 - 6x + x + 3$$

$$y = x^3 + x^2 - 5x + 3$$

so constant $k = 3$

c) Gradient of tangent is equal to gradient of curve

$$y = x^3 + x^2 - 5x + 3$$

$$\Rightarrow \frac{dy}{dx} = 3x^2 + 2x - 5$$

If gradient $= 3$ then

$$3x^2 + 2x - 5 = 3$$

$$3x^2 + 2x - 8 = 0$$

$$3 \times -8 = -24$$

Require factors of -24 that add to give $+2$ $\Rightarrow$ $+6$ and -4

$$3x^2 + 6x - 4x - 8 = 0$$

$$3x(x+2) - 4(x+2) = 0$$

$$(3x-4)(x+2) = 0$$

$$\Rightarrow \underline{x = -2}$$

or

$$3x - 4 = 0$$

$$3x = 4$$

$$\underline{x = \frac{4}{3}}$$

11) AP $a = 30$, $d = -1.5$

a)

$$25^{th} \text{ term} = a + 24d$$

$$= 30 + 24(-1.5)$$

$$= -6$$

b)

$$r^{th} \text{ term} = a + (r-1)d$$

$$30 + (r-1)(-1.5) = 0$$

$$60 - 3(r-1) = 0$$

$$60 - 3r + 3 = 0$$

$$63 = 3r$$

$$r = 21$$

EDEXCEL CORE 1 JAN 2008

11c) S_n largest just before negative terms are added

That is when $n = 21$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$S_{21} = \frac{21}{2}\left(2 \times 30 + 20(-1.5)\right)$$

$$= \frac{21}{2}\left(60 - 30\right)$$

$$= \frac{21}{2} \times 30$$

$$= 21 \times 15$$

$$= 315$$

Questions accidentally omitted

4b) $A(-6, 4)$ $B(8, -3)$

Use Pythagoras to find $|AB|$

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(8 - -6)^2 + (-3 - 4)^2}$$

$$= \sqrt{14^2 + 7^2}$$

$$= \sqrt{196 + 49}$$

$$= \sqrt{245}$$

$$= \sqrt{49 \times 5}$$

$$= 7\sqrt{5}$$

9b) $f'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2}$

At $(4, 1)$ $f'(x) = 4(4) - 6(2) + \frac{8}{16}$

$$f' = 16 - 12 + \frac{1}{2} = \frac{9}{2}$$

$\therefore$ gradient of normal $= -\frac{2}{9}$

Using $y - y_1 = m(x - x_1)$

$$y - 1 = -\frac{2}{9}(x - 4)$$

$$y - 1 = -\frac{2}{9}x + \frac{8}{9}$$

$$y = -\frac{2}{9}x + \frac{17}{9}$$